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1) Compute $\lim_{(x,y)\to(0,0)} f(x,y)$, where $f(x,y) = \frac{2x^2-y^2}{x^2+2y^2}$

Ans If we approach $(0,0)$ along the x-axis, we have $y=0$.

Then $f(x,0) = \frac{2x^2}{x^2} = 2$ for all $x \neq 0$.

Therefore $f(x,y) \longrightarrow 2$ as $(x,y) \longrightarrow (0,0)$ along the x-axis.

If we now approach $(0,0)$ along the y-axis, then $x=0$,
and

$$f(0,y) = \frac{-y^2}{2y^2} = \frac{-1}{2} \text{ for all } y \neq 0.$$

Therefore, $f(x,y) \longrightarrow -\frac{1}{2}$ as $(x,y) \longrightarrow 0$ along the y-axis.

Since f has two different limits along two different lines, $\lim_{(x,y)\to(0,0)} \frac{2x^2-y^2}{x^2+2y^2}$ D.N.E.

2) $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$

Ans Let $f(x,y) = \frac{xy^2}{x^2+y^4}$

Let's approach $(0,0)$ along the y-axis i.e $x=0$.
Then,

$$f(0,y) = \frac{0}{y^4} = 0 \text{ for all } y \neq 0.$$

Therefore,

$$f(x,y) \longrightarrow (0,0) \text{ as } (x,y) \longrightarrow (0,0) \text{ along the } y\text{-axis}.$$

On the other hand, let $(x,y) \longrightarrow (0,0)$ along the parabola $x=y^2$, we have

$$f(x,y) = f(y^2,y) = \frac{y^2 \cdot y^2}{(y^2)^2+y^4} = \frac{y^4}{2y^4} = \frac{1}{2} \text{ for all } y \neq 0.$$

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Therefore, $f(x,y) \longrightarrow \frac{1}{2}$ as $(x,y) \to (0,0)$ along $x = y^2$

Since f has two different limits along two different lines, $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$ D.N.E.

3) Find $\frac{\partial f}{\partial x}$ for $f(x,y) = \frac{ax+by}{cx+dy}$

Soln To find $\frac{\partial f}{\partial x}$, treat y as a constant and differentiate $f(x,y)$ with respect to x.

- $\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\frac{ax+by}{cx+dy}\right) = \frac{(cx+dy).a - (ax+by).c}{(cx+dy)^2}$

$$= \frac{acx + ady - acx - bcy}{(cx+dy)^2} = \frac{y(ad-bc)}{(cx+dy)^2}$$